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Null brane intersections

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Abstract: We study pairs of planar D-branes intersecting on null hypersurfaces, and other related configurations. These are supersymmetric and have finite energy density. They provide open-string analogues of the parabolic orbifold and null-fluxbrane backgrounds for closed superstrings. We derive the spectrum of open strings, showing in particular that if the D-branes are shifted in a spectator dimension so that they do not intersect, the open strings joining them have no asymptotic states. As a result, a single non-BPS excitation can in this case catalyze a condensation of massless modes, changing significantly the underlying supersymmetric vacuum state. We argue that a similar phenomenon can modify the null cosmological singularity of the time-dependent orbifolds. This is a stringy mechanism, distinct from black-hole formation and other strong gravitational instabilities, and one that should dominate at weak string coupling. A by-product of our analysis is a new understanding of the appearance of 1/4 BPS threshold bound states, at special points in the moduli space of toroidally-compactified type-II string theory.

Keywords: D-branes, Superstrings and Heterotic Strings, Cosmology of Theories beyond the SM.

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1. Introduction

The problems of time dependence and cosmology in string theory have recently received considerable attention. A deceptively simple class of time-dependent backgrounds are orbifolds involving boosts or null boosts, rather than spatial rotations [1]–[16]. These are toy models for a cosmological bounce, which is a neccessary ingredient of the pre-Big-Bang [17] and ekpyrotic [18] scenarios. As has been argued however in references [7, 11, 12, 14], strong gravitational effects raises serious questions as to the validity of a perturbative analysis of the spacelike or null singularities in these backgrounds. The fate of such singularities in string theory remains at present an open problem.

A good starting point for addressing these issues [6, 7] is the parabolic orbifold [1] and the associated null flux brane [3]. These closed-string backgrounds are supersymmetric, and have no closed time-like curves. They thus avoid many of the obvious difficulties present in other time-dependent backgrounds. The geometries of interest are orbifolds of flat Minkowski space $\mathbb{R}^{9,1}$ under the action of a Poincaré isometry $\Lambda\mathcal{T}$, consisting of a Lorentz transformation $\Lambda \in \mathrm{SO}(9,1)$, combined with a translation $\mathcal{T}$ that commutes with Λ. The null flux brane is obtained when Λ is a null boost in $\mathrm{SO}(2,1)$ and $\mathcal{T}$ a translation by $2\pi r$ in the remaining $\mathbb{R}^7$. The parabolic orbifold is the special case $r = 0$. The null flux brane is a non-singular geometry, which does not suffer from the strong-coupling instabilities of [12, 14] for given r and sufficiently weak string coupling. To a fundamental string probe, on the other hand, a flux brane with $r \sim \sqrt{\alpha'}$ should look indistinguishable from the parabolic orbifold background. Thus there is legitimate hope that one may understand the nature of the null singularity within string perturbation theory in this simple model.

Motivated by such questions, we analyze in the present work an open-string analogue of the parabolic orbifold and null flux brane backgrounds. The system consists of two planar D-branes of type-II string theory, whose worldvolumes are related by the Poincaré transformation $\Lambda\mathcal{T}$. The two branes are in relative motion and make a non-zero angle with each other, and the minimal distance between them is $2\pi r \equiv b$. When $b = 0$ they intersect on a null hypersurface, which plays a similar role to the null singularity in the parabolic orbifold. A simple example of such a background has two linear D-strings oriented and moving so that the point at which they intersect propagates at the speed of light. We call this the null-scissors configuration, and its generalisation to non-zero b will be referred to as the shifted null scissors. Such configurations preserve 1/2 of the supersymmetries of the individual branes,[1] for the same algebraic reason that determines that the parabolic orbifold and flux brane preserve 1/2 of the 32 supersymmetries of flat spacetime. Furthermore, open strings with one end on each D-brane are directly analogous to the twisted closed strings of[7, 13] in the corresponding orbifold backgrounds. Open strings with both ends on the same D-brane correspond likewise to untwisted closed strings.

One of the main messages of our work is that twisted closed strings and their open-string counterparts will trigger an instability of the underlying vacuum, if produced during a collision process. The reason, as we will show, is that these strings have no normalizable asymptotic states unless $r = 0$ in the orbifold, or $b = 0$ for the D-branes. In the shifted null scissors configuration, this mechanism will force the D-strings to intersect, and then recombine as in a standard string interaction. We expect similar phenomena in the null flux brane background, for $r \sim \sqrt{\alpha'}$ and sufficiently weak string coupling constant. In this regime there will be no formation of black holes [14], and the above stringy mechanism could dominate. Ultimately, one would of course like to have a good effective description of the null D-brane intersection, and of the related null Big-Crunch/Big-Bang singularity, at weak string coupling. It could be that such a description must be of a statistical nature. Our analysis in this paper is aimed at identifying the relevant modes, and should be considered as a step in this direction.

The plan of this paper is as follows: in section 2 we introduce the basic null-scissors and shifted null-scissors configurations of type-II string theory, and compare them to the more familiar cases of static branes at angles, and of parallel but moving branes. In section 3 we describe various dual configurations, and explain why they are 1/4 supersymmetric. We discuss in detail a particular T-dual configuration, consisting of a pair of static parallel D3-branes, one of which carries an infinite-wavelength plane electromagnetic wave on its worldvolume. The analogs of twisted (untwisted) closed strings are open strings that are charged (neutral) with respect to the electromagnetic field. In section 4 we analyze the open-string excitations on the D-branes. Adapting the discussion of reference [7], we show that the only normalizable states of the charged open strings have their momenta pointing in the direction of the electromagnetic wave and are massless. Such states thus only exist for zero shift, or for $r = 0$ in the closed-string background. In section 5, we wrap the null-intersecting D-strings on a torus, which is possible only for special values of the

[1] This is shown in reference [19], which contains a general analysis of unbroken supersymmetries for pairs of branes related by an arbitrary element of the Lorentz group SO(9, 1).

torus moduli, and show that the two D-strings can form a supersymmetric bound state at threshold. This sheds new light on the appearance of 1/4 BPS threshold bound states at special points in the moduli space of toroidally compactified type-II strings. Finally, in section 6 we turn to the important issue of stability. We first discuss supersymmetric generalizations of the null D-brane scissors, in which the D-strings carry arbitrary waves, all travelling in the same direction as the intersection point. We then explain how a low-energy collision process can trigger a condensation of massless modes, leading to a modification of the vacuum that corresponds to the splitting and joining of the two D-strings. We comment on the analogous orbifold problem and suggest directions for future work.

2. Null scissors

Consider the configuration of figure 1. An infinite straight rigid rod makes an angle θ with a 'reference' rod, and moves with uniform velocity v in the normal (downward) direction. The reference rod is at rest and extends along the x^1 axis, while the other moves in the (x^1, x^2) plane. As can be verified easily, the intersection point (I) of these two rods propagates with velocity $v_{\mathrm{I}} = v/\sin\theta$ in the (negative) x^1 direction. There are thus three inequivalent possibilities: for v greater, less, or equal to $c\sin\theta$, the intersection velocity is greater, less, or equal to the speed of light.

For a given v and sufficiently small angle θ, we have $v_{\mathrm{I}} > c$ (henceforth we set $c = 1$). In books on special relativity this is sometimes called the scissors[2] paradox [20]. There isn't of course anything paradoxical in this example: the superluminal propagation of the intersection point cannot be used to send a signal faster than light. One could for instance attach one end of a string to one of the rods and the other end to the other rod and let the string move down with the intersection point. However, rather than keep pace with this latter, the string will either break or bend the rods, and slow or stop their relative motion.

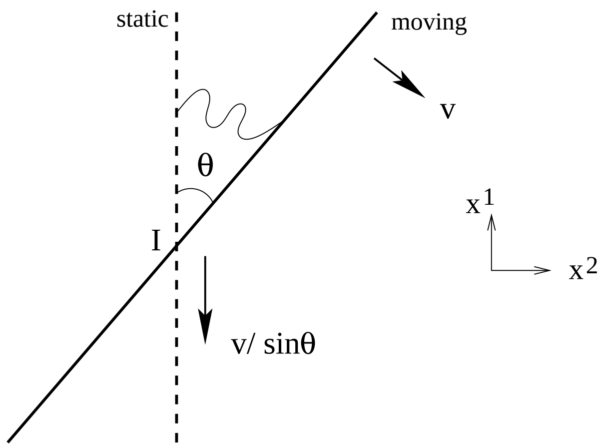

Figure 1: A D1-brane, rotated by an angle θ relative to the x^1 axis, and moving in the normal direction with uniform velocity v. Its intersection with a static reference D1-brane propagates along the negative x^1 direction with speed $v_{\mathrm{I}} = v/\sin\theta$.

[2]This is a slight misnomer because in common scissors the intersection point does not move.

The above configuration can be realized in type-IIB string theory, where the rods may be infinite straight D1-branes. We assume for the moment that both D1-branes sit at $x^3 = \cdots = x^9 = 0$. In the subspace spanned by x^0, x^1, x^2 their worldvolume embeddings are described by the following equations:

$$\begin{aligned} &\textit{static}: \quad x^2 = 0\,, \\ &\textit{moving}: \quad x^2 = \frac{(v\,x^0 + \sin\theta\, x^1)}{\cos\theta}\,, \end{aligned} \tag{2.1}$$

while their intersection is given by

$$\textit{intersection}: \quad x^2 = x^1 + \frac{v}{\sin\theta}\, x^0 = 0\,. \tag{2.2}$$

As already stated, the intersection is timelike, spacelike, or null for v respectively smaller, greater, or equal to $\sin\theta$.

If the intersection trajectory is timelike, we can bring the point I (as well as the moving D1-brane) to rest, by boosting with velocity $v/\sin\theta < 1$ in the x^1 direction. As the boost is in a longitudinal direction, the static brane is left unchanged. The transformed configuration has two static D1-branes, intersecting at an angle

$$\theta' = \arctan\left(\frac{\sqrt{\sin^2\theta - v^2}}{\cos\theta}\right). \tag{2.3}$$

This is a non-supersymmetric and unstable configuration, in which the lowest-lying state of stretched open strings joining the two branes will be tachyonic (see for instance [21]).

If the intersection trajectory is spacelike, it can be brought to the special form $x^{2\,\prime} = x^{0\,\prime} = 0$. This can be achieved by a Lorentz boost with velocity $\sin\theta/v < 1$ in the x^1 direction. The boost again leaves the static brane invariant. The transformed configuration thus has two parallel D1-branes, moving with relative speed

$$v' = \frac{\sqrt{v^2 - \sin^2\theta}}{\cos\theta}\,, \tag{2.4}$$

and coinciding at $x^{0\,\prime} = 0$. This is again a non-supersymmetric configuration, with velocity-dependent forces and an instability for pair creation of stretched open strings [22, 23].

The third case, that of a null worldvolume intersection, is the one that will interest us here. It cannot be transformed to a static configuration, nor to one of moving but parallel D-branes. The two D-brane worldvolumes are in this case related by the 'null Lorentz transformation'[3]

$$\Lambda(\theta) = \exp(-\sqrt{2}\,\tan\theta\,\mathcal{J}) \qquad \text{with } \mathcal{J} = \frac{i}{\sqrt{2}}(J^{\,02} + J^{12})\,. \tag{2.5}$$

In the basis $\mathbf{x} = (x^+, x^2, x^-)$, with $x^\pm = (x^0 \pm x^1)/\sqrt{2}$, the generator $\mathcal{J}$ takes the simple form

$$\mathcal{J} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{2.6}$$

[3]We use the conventions $J^{\mu\nu} = -i(x^\mu\partial^\nu - x^\nu\partial^\nu)$ and $\eta^{\mu\nu} = (-++\ldots+)$.

The reader can easily verify that the condition $y^2 = 0$, with $\mathbf{y} = \Lambda(\theta)\,\mathbf{x}$, is the same as the equation (2.1) which defines the embedding of the moving D1-brane, in the critical case $v = \sin\theta$. We will show in the following section that, in contrast to the two other cases which are non-supersymmetric, the null-scissors configuration is 1/4 supersymmetric.

A simple generalization that still preserves 1/4 spacetime supersymmetry is one in which the moving D1-brane is displaced in the x^3 direction by a distance b, so that the reference brane is at $x^3 = x^4 = \cdots = x^9 = 0$ while the moving one is at $x^3 = b$, $x^4 = \cdots = x^9 = 0$. We will refer to this configuration as the 'shifted null scissors'. The intersection point is here replaced by the shortest linear segment joining the D-strings. This has length b and moves with speed $v_{\rm I} = 1$. It is important to stress that b is the only physical parameter of this setup. Indeed, the velocity $v = \sin\theta$ can be given any value between 0 and 1 by Lorentz boosting in the x^1 direction. We will later choose for convenience

$$\tan\theta = \frac{\pi}{\sqrt{2}}\,, \qquad \text{so that } \Lambda(\theta) = e^{-\pi\mathcal{J}}\,. \tag{2.7}$$

The boost required to go to this special frame does not of course affect the intersection worldline, which remains null.

The 'null scissors' and 'shifted null scissors' configurations are the open-string analogues of the parabolic orbifold [1] and null flux-brane [3] backgrounds for closed strings. The generator of the orbifold group in these backgrounds is the same as the Poincaré transformation that relates the static and moving branes in our setting. Part of our motivation for the present work was to gain more insight into the physics of the corresponding closed-string problem. We will return to this relationship later on.

3. Duality maps and supersymmetry

The null-scissors configuration can be transformed to other equivalent configurations by (chains of) duality maps. For instance, the following series of dualities:

$$D1 \xrightarrow{T} D5 \xrightarrow{S} NS5_B \xrightarrow{T} NS5_A \xrightarrow{\text{lift}} M5\,, \tag{3.1}$$

maps the two D-strings of type-IIB theory to two M-theory fivebranes, which intersect along a five-dimensional null hypersurface. The first T-duality acts in four transverse dimensions, say (3456), while the T-duality between the type-IIB and type-IIA theories only acts on one of these four dimensions. The final M-theory configuration is precisely the one considered by the authors of reference [19].

Following [19], let us verify that the above configurations leave 1/4 of the 32 supersymmetries unbroken. The Killing spinors in the case of M5-branes have constant asymptotic values ϵ that must obey

$$\mathbf{\Pi}\epsilon = \tilde{\mathbf{\Pi}}\epsilon = \epsilon\,. \tag{3.2}$$

Here

$$\mathbf{\Pi} = \gamma^{013456} \qquad \text{and} \qquad \tilde{\mathbf{\Pi}} = S(\theta)\,\mathbf{\Pi}\,S(\theta)^{-1}\,, \tag{3.3}$$

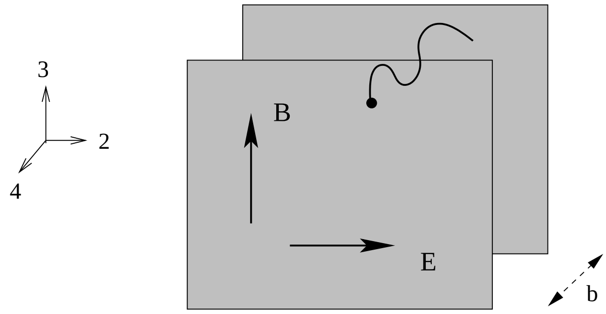

Figure 2: A T-dual configuration of the null scissors of figure 1. It is obtained by T-dualizing the x^2 and x^3 dimensions. One of the two resulting D3-branes carries equal in magnitude, but orthogonal electric and magnetic fields. The branes can be separated by a distance b in the x^4 dimension.

with $S(\theta)$ the null boost (2.5) in the spinor representation of $O(1,10)$. If $S(\theta)$ were a pure boost, or a simple rotation on a plane, conditions (3.2) would have no solution. In our case, using $J^{\mu\nu} = -\frac{i}{2}\gamma^{\mu\nu}$ and $(\gamma^+)^2 = 0$, one finds

$$S(\theta) = \exp\left(\frac{\tan\theta}{\sqrt{2}}\gamma^{2+}\right) = 1 + \frac{\tan\theta}{\sqrt{2}}\,\gamma^{2+}\,. \tag{3.4}$$

It follows that both conditions can be simultaneously satisfied by spinors that obey the chiral projections

$$\gamma^{01}\epsilon = \gamma^{3456}\epsilon = \epsilon\,. \tag{3.5}$$

These leave precisely 8 unbroken supersymmetries. All dual configurations are of course also 1/4 supersymmetric, as are the shifted cases in which one of the branes has been displaced in an orthogonal direction.

We can gain some further insight into the null scissors of figure 1 by compactifying the x^2 dimension on a circle, and then performing a T-duality transformation. This inverts the value of the radius in string units, transforms the D1-branes to D2-branes, and maps their embedding coordinates to Wilson lines (see e.g. [23]). One finds a gauge potential $A^2 = 0$ for the D2-brane dual to the static string, and

$$2\pi\alpha' A^2 = \frac{v}{\cos\theta}x^0 + \tan\theta\, x^1 \tag{3.6}$$

for the D2-brane dual to the moving one. The final configuration is illustrated in figure 2. It has a pair of static parallel D2-branes, one of which carries a constant worldvolume electromagnetic field. For the shifted scissors, in which one of the D-strings is displaced by a distance b in the x^4 direction, the dual D2-branes are also separated by a distance b along x^4. Further T-dualities along spectator directions would add extra dimensions to the D2-branes, without affecting the electromagnetic field.

The three different cases, of timelike, spacelike or null intersection, can be easily seen to correspond to positive, negative or zero values of the invariant quantity $F_{ab}F^{ab}$. In the first two of these cases, a Lorentz boost can make the field either pure magnetic, or pure electric. This is impossible in the null case, where the field reads

$$F_{+2} = \frac{\tan\theta}{\sqrt{2}\pi\alpha'}\,, \qquad \text{all other } F_{ab} = 0\,. \tag{3.7}$$

We can think of this background as the infinite-wavelength limit of a plane electromagnetic wave. Of course, since a Lorentz boost can rescale F_{+2}, the only invariant statement (assuming non-compact branes) is that the parameter $\tan\theta$ is finite and non-zero.

We will see later, in section 6, that one can consider general electromagnetic waves with arbitrary profiles $F_{+2}(x^+)$. These will be T-dual to configurations in which the D-branes carry transverse-displacement waves, all travelling in the same direction at the speed of light [24]–[29]. Our straight null scissors can be obtained as a special limit of such general configurations, all of which are 1/4 supersymmetric.

The above T-duality transformation gives an interesting interpretation [22] of the instability of electric fields in string theory [30]. An electric field is classically unstable in the presence of charged particles, which accelerate and pump energy out of the field. This is T-dual to the phenomenon whereby an open string joining the two D-branes cannot keep pace with their intersection point if this moves with superluminal speed. The string will stretch, pumping energy out of the moving branes and bringing them eventually to rest. Furthermore, even the open-string vacuum state is quantum-mechanically unstable by pair production of such stretching strings, a phenomenon T-dual to the well-known Schwinger instability of electric fields.

4. Open-string spectrum

In this section we analyze the open-string theory for the null D-brane scissors, and for its T-dual configurations. There are three types of open strings in the configuration of figure 1: those living on the static D-brane, those living on the moving D-brane, and those stretching from the static to the moving one. In the T-dual picture of figure 2, the latter correspond to strings charged under the electromagnetic background fields, while the former two types of string are neutral.

Strings with both endpoints on the same D-brane have a standard spectrum, consisting of a maximally-supersymmetric spin-1 multiplet, plus the usual tower of massive excitations. The only minor subtlety has to do with worldsheet zero modes. The strings on the static D-brane of figure 1 have momentum $p^2 = 0$,[4] while those on the moving brane have $p^2 = \sqrt{2}\,\tan\theta\, p^+$. Thus, the mass-shell condition for the latter reads

$$-2p^+p^- + (\sqrt{2}\,\tan\theta\, p^+)^2 + (p_\perp)^2 + M^2 = 0\,, \tag{4.1}$$

where $p_\perp$ is the momentum in the spectator dimensions $(3,\ldots,9)$, along which the D-brane may possibly extend. Solving condition (4.1) gives p^- in terms of p^+, $p_\perp$ and the mass M. It is important here to realize that only p^+ and $p_\perp$ are conserved momenta in the

[4]Note that here and in what follows, p^2 is the $\mu = 2$ component of p^μ, and x^2 is the $\mu = 2$ component of x^μ. The magnitude squared of the vectors will be written as $p_\mu p^\mu$ and $x_\mu x^\mu$.

interacting theory. Neither p^- nor p^2 will be conserved in general, since translations of x^+ and x^2 are not symmetries of the D-brane background. These symmetries will be violated by open-string diagrams whose boundary has components on both D-branes.

It is interesting to see how condition (4.1) will arise in the T-dual picture of figure 2. The fact that the strings in figure 1 do not wind in the x^2 direction becomes the condition in the T-dual picture that $p^2 = 0$. In the presence of a background F field, open strings feel the effective metric [31]

$$G_{ab} = \eta_{ab} + (2\pi\alpha')^2 F_{ac}F_{bd}\,\eta^{cd}\,. \tag{4.2}$$

Here a, b are worldvolume indices, which are identified with a subset of spacetime indices in static gauge. Inserting (3.7) in this expression we find

$$ds^2_{\text{open}} = \eta_{ab}\,dx^a dx^b + 2\,\tan^2\theta\,(dx^+)^2\,. \tag{4.3}$$

It is now easy to verify that the mass-shell condition

$$G_{ab}\,p^a p^b + M^2 = 0 \tag{4.4}$$

is the same as eq. (4.1), provided we set $p^2 = 0$. The more general case, $p^2 \neq 0$, should be compared to strings with non-zero winding w^2 in the T-dual scissors.

We turn next to the study of open strings with one endpoint on each of the two D-branes. These will play a crucial role in our discussion of stability in section 6. They are, as will become clear, the open-string analogues of the twisted closed strings of references [7, 12, 13]. We will in fact simply adapt the analysis of these references to our problem.

We will consider the configuration of figure 2, and limit our discussion to the bosonic coordinates. Transforming back to the null scissors via a T-duality, and extending the analysis to worldsheet fermions, are simple matters of detail which we will skip here. The boundary condition at the endpoint of an open string coupled to a constant electromagnetic background is

$$\partial_\sigma X^a = \pm 2\pi\alpha'\, F^a{}_b \partial_\tau X^b\,. \tag{4.5}$$

The sign depends on the orientation. Using equations (3.7) and (2.7), and the definition (2.6) of the matrix $\mathcal{J}$, we find the following conditions for a stretched open string:

$$\partial_\sigma \mathbf{X} = 0 \quad \text{at } \sigma = 0\,, \qquad \text{and} \qquad \partial_\sigma \mathbf{X} = \pi\mathcal{J}\,\partial_\tau \mathbf{X} \quad \text{at } \sigma = \pi\,. \tag{4.6}$$

We use the 3-component vector notation $\mathbf{X} = (X^+, X^2, X^-)$, and we have dropped the seven 'spectator' coordinates which obey regular Dirichlet or Neuman conditions at both string endpoints.

The general expression for harmonic coordinates with the boundary conditions (4.6) can be written using the 'spectral-flow' trick of [7] as follows:

$$\mathbf{X}(\tau,\sigma) = \mathbf{X}_0(\tau,\sigma) + \sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}(\mathcal{J} - in)^{-1}\left[\,e^{(\mathcal{J}-in)(\tau+\sigma)} + e^{(\mathcal{J}-in)(\tau-\sigma)}\right]\mathbf{a}_n\,. \tag{4.7}$$

Here $\mathbf{a}_n$ is a triplet of oscillation amplitudes, obeying the reality conditions $\mathbf{a}_{-n} = \mathbf{a}_n^*$. The zero-mode piece is given by

$$\mathbf{X}_0(\tau,\sigma) = \mathbf{x}_0 + \sqrt{\frac{\alpha'}{2}}\,\mathbf{f}(\tau+\sigma) + \sqrt{\frac{\alpha'}{2}}\,\mathbf{f}(\tau-\sigma)\,, \tag{4.8}$$

with

$$\mathbf{f}(y) = \int_0^y dw\; e^{\mathcal{J}w}\mathbf{a_0}\,. \tag{4.9}$$

To check that (4.7) indeed satisfies the boundary conditions (4.6) one must use the fact that $\mathcal{J}$ is nilpotent. It obeys the equation $\mathcal{J}^3 = 0$, which implies in particular that $\tanh(\pi\mathcal{J}) = \pi\mathcal{J}$. Using the nilpotency of $\mathcal{J}$ we can also write (4.9) as follows:

$$\mathbf{f}(y) = y\;\mathbf{a_0} + \frac{y^2}{2}\,\mathcal{J}\mathbf{a_0} + \frac{y^3}{6}\,\mathcal{J}^2\mathbf{a_0}\,. \tag{4.10}$$

The reader can check that in the formal limit $\mathcal{J}\to 0$, expression (4.7) reduces to the standard expansion [32] for Neumann coordinates, as expected.

To solve for the classical motions of the open string, we need to impose the conformal-gauge conditions. The Virasoro generators have the usual form in terms of the oscillation amplitudes:

$$L_n = \frac{1}{2}\sum_{m=-\infty}^{\infty} \mathbf{a}_{n-m}\cdot\mathbf{a}_m\ , \tag{4.11}$$

where the dot denotes the lorentzian inner product, and we must here put back the spectator dimensions in the definition of the vectors $\mathbf{a}_n$. Equation (4.11) follows from the fact that $e^{x\mathcal{J}}$ is a Lorentz transformation, so that

$$(e^{x\mathcal{J}})^\mu{}_\rho\,(e^{x\mathcal{J}})^\nu{}_\sigma\,\eta^{\rho\sigma} = \eta^{\mu\nu}\,.$$

Now since

$$p^+ = \frac{1}{2\pi\alpha'}\int_0^\pi d\sigma \dot{X}^+ = \frac{a_0^+}{\sqrt{2\alpha'}} \tag{4.12}$$

is a conserved momentum, we can go to the light-cone gauge where $X^+ = 2\alpha' p^+\tau$. We can then solve the Virasoro conditions, $L_n = 0$, by expressing the a_n^- in terms of the remaining independent amplitudes a_n^j.

To gain some insight into these classical solutions, let us assume that the string does not oscillate in the direction x^2, so that $a_n^2 = 0$ for $n \neq 0$. The center of mass momenta, defined as in equation (4.12), are in this case:

$$\mathbf{p}(\tau) = \frac{1}{\sqrt{2\alpha'}}\begin{pmatrix} a_0^+ \\ a_0^2 + a_0^+\tau \\ a_0^- + a_0^2\tau + a_0^+(\frac{\tau^2}{2}+\frac{\pi^2}{6}) \end{pmatrix}. \tag{4.13}$$

If $a_0^+ = \sqrt{2\alpha'}p^+$ does not vanish, p^2 and p^- grow in time, as does the total energy $p^0 = (p^+ + p^-)/\sqrt{2}$. These growing modes draw their energy from the electromagnetic background, and this solution is only reliable until the energy grows so large that the back-reaction can no longer be ignored (the effect of the back-reaction will be considered in section 6). Furthermore, a simple calculation shows that

$$-p^\mu p_\mu = \frac{(\pi p^+)^2}{3} + M_\perp^2 \,, \tag{4.14}$$

where

$$M_\perp^2 = \frac{1}{\alpha'} \sum_{j=3}^{9} \sum_{n=1}^{\infty} |a_n^j|^2 + \left(\frac{b}{2\pi\alpha'}\right)^2 \tag{4.15}$$

is the mass due to transverse oscillations, and to the shift b. Equation (4.14) shows that the total mass of the string does not grow, and the absorbed energy is kinetic rather than tension energy. This is not the case in the equivalent configuration of figure 1, where strings with $p^+ \neq 0$ will stretch and so have both tension and kinetic energy. As a result the effective mass must grow with time. The reader can check these claims explicitly, by performing the T-duality transformation which flips the sign of the right-moving component of X^2 in (4.8).

When $p^+ = 0$, the string travels on the lightcone and it must therefore be massless. The general solution, up to reparametrizations,[5] reads

$$X^- = \sqrt{2\alpha'}\, p^- \tau \,, \quad X^+ \quad \text{and} \quad X^j \qquad \text{constant} \,. \tag{4.16}$$

It describes a zero-size massless string moving in the direction normal to the electric and magnetic fields at the speed of light. In the T-dual null scissors of figure 1 the string is localized at the intersection ($X^+ = X^2 = 0$). Clearly, such a solution does not exist in the shifted null-scissors configuration, or its dual versions. The string must stretch in a spectator dimension in the shifted case, and is therefore necessarily massive. To summarise our conclusions thus far: sustainable classical string motions are only possible in the $b = 0$ case, and they have $p^+ = p^i = M_\perp = 0$, and arbitrary p^-. There are also classical growing modes, which can signal an instability, as will be discussed in section 6.

Consider now the quantized theory. Canonical commutation relations between $X^\mu(\sigma)$ and the conjugate variables $\Pi_\mu(\sigma)$ imply the following commutators between the oscillator amplitudes and the zero modes in the expansion (4.7) – (4.9):

$$[a_m^\mu, a_n^\nu] = (m\eta^{\mu\nu} - i2\alpha'\, F^{\mu\nu})\delta_{m+n,0} \,, \qquad \text{and} \qquad [x_0^\mu, a_0^\nu] = i\sqrt{2\alpha'}\eta^{\mu\nu} \,. \tag{4.17}$$

If we define $p^\mu \equiv a_0^\mu/\sqrt{2\alpha'}$ (these are the classical center-of-mass momenta at $\tau = 0$), then the algebra of zero modes reads:

$$[x_0^\mu, p^\nu] = i\eta^{\mu\nu} \qquad \text{and} \qquad [p^-, p^2] = if \,, \qquad \text{where } f \equiv 2\alpha' F^{2-} \,. \tag{4.18}$$

This is an unfamiliar algebra, that we derive in the point-particle limit in the appendix. We can, in fact, easily get rid of the 'anomalous' commutator by redefining

$$p^- \implies p^- - fx^2 \,. \tag{4.19}$$

[5] If X^+ were not constant, we could find a gauge such that $X^+ \propto \tau$ in a local patch. By continuity this would then reduce to one of the previously discussed solutions.

This does not affect the remaining commutators, but it modifies the zero-mode contribution to the Virasoro generator L_0,

$$p^\mu p_\mu \implies p^\mu p_\mu + 2p^+ f x^2 \,, \tag{4.20}$$

and hence also the corresponding physical-state condition.

There are two distinct situations: $p^+ = 0$ and $p^+ \neq 0$. In the first case the wave operator (4.20) has the standard form, so that massless states with $p^j = 0$ are physical. These are the states in the vector supermultiplet of the open superstring,[6] travelling in the same direction as the intersection point. For $p^+ \neq 0$, on the other hand, the wave operator is that of a particle moving in a potential that varies linearly with a spatial dimension. This problem has no normalizable eigenmodes, so the superstring will have no normalizable physical states in this case. Physical quantum states are thus in one-to-one correspondence with sustainable classical motions.

The mode expansion and oscillator algebra for charged open strings are closely related to those for the twisted strings in the parabolic orbifold and null-fluxbrane backgrounds of references [7, 13]. As in these references, one can formally construct vertex operators corresponding to a tower of massive string states. However, since the corresponding wave-functions are not normalizable, these states are not really asymptotic, i.e. they cannot appear as external legs in string amplitudes. As we will argue later, in section 6, they must decay to massless $p^+ = 0$ modes for zero shift, and they can catalyze a condensation of zero modes when $b \neq 0$.

5. Threshold bound states

We have assumed so far that the null scissors are made out of two infinite D-strings. In this section we will discuss what happens when (x^1, x^2) are coordinates on a flat two-dimensional torus. The angle θ and the velocity v must satisfy separate quantization conditions in this case, so the requirement that $v = \sin\theta$ imposes a restriction on the moduli of the torus. As we will now show, on this special locus of the torus moduli space the two D-strings can form a bound state at threshold, so that their own classical moduli space has both a Coulomb and a Higgs branch.

To simplify the discussion, consider an orthogonal torus with radii R_1 and R_2. Let one of the D-strings stretch in the x^2 direction and move in the orthogonal direction x^1, while the second string is taken oblique and static, as illustrated on the left-hand side of figure 3. If the moving string winds m_2 times around the x^2 dimension, momentum quantization implies that

$$\frac{2\pi m_2 R_2 T\, v}{\sqrt{1-v^2}} = \frac{n_1}{R_1} \,, \tag{5.1}$$

with T the tension of a D-string. Furthermore, if the static string winds l_1 and l_2 times in the x^1 and x^2 dimensions, then the angle θ with the $O2$ axis is given by

$$\tan\theta = \frac{l_1 R_1}{l_2 R_2} \,. \tag{5.2}$$

[6] It is straightforward to verify that the tachyon subtraction and polarization conditions are the usual ones.

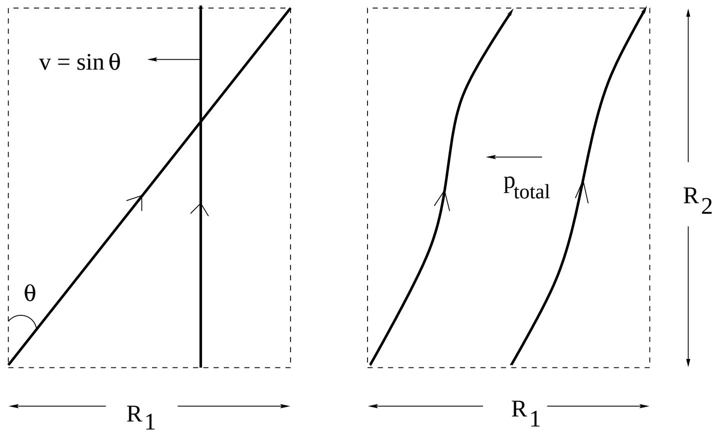

Figure 3: Two D-strings related by a null boost and compactified on a torus (left). The strings can form a bound state at threshold, represented by a single classical D-string carrying the total winding and momentum numbers of the pair (right). The longitudinal momentum is carried by transverse oscillation waves.

Combining equations (5.1) and (5.2) with the null-boost condition $|v| = |\sin\theta|$ we find

$$2\pi T R_1^2 = \left|\frac{n_1 l_2}{m_2 l_1}\right| . \tag{5.3}$$

This fixes one of the torus radii in terms of the quantum numbers of the two D-strings. Note that if we hold the winding numbers, radii and fundamental-string scale fixed, and take the string coupling g_s to zero, then $T \sim n_1 \sim 1/g_s$.

Consider now a single D-string carrying the sum of the charges of the above pair, i.e. winding $(l_1, l_2 + m_2)$ times around the two circles, and carrying n_1 units of momentum in the direction $O1$. This is illustrated, for the special case $l_1 = l_2 = m_2 = 1$, on the right-hand side of figure 3. For generic values of the radii, the composite string is a 1/4 BPS subthreshold bound state of its two components. We will show that when condition (5.3) is satisfied, the binding energy is precisely zero. To this end, it is convenient to use the S-duality of the type-IIB theory, and transform the D-strings to fundamental strings. The latter are described by $O(2,2)$ charge vectors

$$q \equiv (\vec{q}_L, \vec{q}_R) \equiv (\vec{p} + \vec{w}, \vec{p} - \vec{w}) , \tag{5.4}$$

where $\vec{w}$ and $\vec{p}$ are the winding and momentum vectors. In the case at hand we have (we use here units such that $2\pi T = 1$):

$$\begin{aligned}
&\textit{moving}: \quad \vec{q}_L = \left(\frac{n_1}{R_1}, m_2 R_2\right) , \quad \vec{q}_R = \left(\frac{n_1}{R_1}, -m_2 R_2\right) ,\\
&\textit{static}: \quad \vec{q}_L^{\,\prime} = -\vec{q}_R^{\,\prime} = (l_1 R_1, l_2 R_2) .
\end{aligned} \tag{5.5}$$

The charge vector of the bound state is equal to the sum $q + q'$, while its mass in the ground state is given by

$$M = \max(|\vec{q}_L + \vec{q}_L^{\,\prime}|, |\vec{q}_R + \vec{q}_R^{\,\prime}|) . \tag{5.6}$$

The binding energy is $\mathcal{E} = |\vec{q}_L| + |\vec{q}_L^{\,\prime}| - M = |\vec{q}_R| + |\vec{q}_R^{\,\prime}| - M$ and for this to vanish, either $\vec{q}_L^{\,\prime}$ must be parallel to $\vec{q}_L$, or $\vec{q}_R^{\,\prime}$ must be parallel to $\vec{q}_R$. The reader will easily verify that this condition is equivalent to (5.3).

We can further generalize the discussion, so as to allow for arbitrary momenta to flow on the static and the moving branes of figure 3. Let us T-dualise in a spectator dimension to the type-IIA theory, where one can recognize more readily the conditions for unbroken supersymmetry of a fundamental string,

$$(M\gamma^0 - \vec{q}_L \cdot \vec{\gamma})\epsilon_L = 0 \qquad \text{and} \qquad (M\gamma^0 - \vec{q}_R \cdot \vec{\gamma})\epsilon_R = 0\,. \tag{5.7}$$

The states (5.5) have $M = |\vec{q}_L| = |\vec{q}_R|$ and $M' = |\vec{q}_L^{\,\prime}| = |\vec{q}_R^{\,\prime}|$, so both of the above conditions have non-trivial solutions. Each of these states is thus 1/2 BPS, while together they only preserve 1/4 of the 32 supersymmetries. We have already seen this in the dual configuration of section 3, but let us check it again in the present context. For strings oriented and moving as in figure 3, n_1 is a negative integer while l_1, l_2 and m_2 are positive. Using equation (5.3) one can check that the vectors $\vec{q}_R$ and $\vec{q}_R^{\,\prime}$ point in the same direction, while for non-zero angle θ, $\vec{q}_L$ and $\vec{q}_L^{\,\prime}$ are not aligned. Thus, only one half of the ϵ_R Killing spinors survive simultaneously the projections (5.7) for the two strings on the left of figure 3. Now adding momentum $\vec{p}^{\,\prime}$ in the $-\vec{w}^{\,\prime}$ direction (parallel to $\vec{w}^{\,\prime}$ and in the oppoiste direction) on the static string does not break any further supersymmetries. This is because $\vec{q}_R^{\,\prime}$ will not change its direction, and $|\vec{q}_R^{\,\prime}|$ is (strictly) bigger than $|\vec{q}_L^{\,\prime}|$. Our two strings can thus still form a threshold bound state in this case. Clearly, by symmetry, we may also add momentum on the moving string. Notice that these momenta on the individual strings point in the direction of the intersection's motion. Any other small perturbation of the strings or torus would lead to a sub-threshold bound state.

To summarise, we have shown in this section that 1/2 BPS branes of type-II string theory, related by a null boost, can form threshold bound states when compactified on a torus. This remains true even in the presence of massless excitations, provided these latter propagate in the same direction as the intersection of the branes (or as the shortest linear segment between them, for non-zero shift). Though this conclusion is ultimately a consequence of supersymmetry, it does give a nice alternative and intuitive explanation for the appearance of 1/4 BPS threshold bound states, at special points in the moduli space of the toroidally-compactified type-II string theory.

6. Deformations, stability and the parabolic orbifold

A configuration with 8 unbroken supersymmetries, like the null D-brane scissors of section 2, is normally expected to be stable. This is not, however, necessarily the relevant question. Our D-brane configurations have a classical moduli space, and one would like to understand whether small perturbations (such as open-string excitations) can push us far from the original vacuum state. We will now argue that this is indeed the case, and that a small perturbation can lead to a condensation of open strings that modifies the underlying vacuum state. Put differently, though quantum fluctuations do not destabilize the open-string vacuum, there can still be significant back-reaction in the one- or multi-particle states. A similar conclusion has been reached for the parabolic orbifold in [7, 11, 12, 14]. The instability discussed here is however different: it is a stringy phenomenon, rather than a strong-gravity effect, and it a priori relevant for the smooth flux-brane with $r \sim \sqrt{\alpha'}$ even at vanishingly-small string coupling.

The basic process can be described as follows: consider a fundamental open string moving in from spatial infinity towards the intersection (or the nearest-distance) point with both its ends attached to one of the two D-branes. Such a string must have strictly positive p^+, or else it will be co-moving with the nearest-distance point. Furthermore, since the unbroken supersymmetries are chiral ($\gamma^+\epsilon = 0$) the string is a non-BPS excitation of the vacuum state. Now in the vicinity of the nearest-distance point, our incoming excitation can interact and produce a pair of open strings stretched between the two D-branes, which carry some or all of the conserved momentum p^+. The probability that this will happen is exponentially small for large shift b, but it should be of order one when $b \sim \sqrt{\alpha'}$. What is the fate of the D-brane scissors if this happens?

Since the stretched open strings carry Chan-Paton charge, they must either annihilate in pairs or else decay to their least-excited state. Generically, the strings will separate in space along $O1$ (and possibly along other spectator dimensions), so they cannot annihilate in pairs. On the other hand, as we have seen in section 4, an isolated stretched string has no normalizable states for non-zero shift. The string will therefore accelerate and grow, pumping energy out of the pair of moving D-branes. The only possible outcome in the end, is that back-reaction forces the pair of D-branes to intersect, so that the stretched open strings can then decay to one of their normalizable $p^+ = p^j = M_\perp = 0$ states.

To better understand this process, let us consider all marginal deformations of the D-brane scissors which respect the 8 unbroken supersymmetries. They correspond to massless open string states with $p^+ = p^j = 0$, and any p^-. Turning on these marginal operators on a single D-brane amounts to introducing travelling waves, with a profile that is an arbitrary function of x^+. Such D-branes are known [25, 28, 29] to be 1/4 BPS states of type-II string theory — indeed we have already encountered their dual version in section 5. To be specific, the following generalization of the embeddings (2.1) is compatible with eight spacetime supersymmetries:

$$\begin{aligned} &\textit{brane } 1: \quad x^j = Y^j(x^+)\,, \\ &\textit{brane } 2: \quad x^j = \tilde{Y}^j(x^+)\,, \end{aligned} \tag{6.1}$$

where the Y^j and $\tilde{Y}^j$ are arbitrary functions of the light-cone time x^+ (see figure 4). T-dualizing all transverse dimensions gives two D9-branes carrying electromagnetic waves, with arbitrary profiles $F_{+j}(x^+) = dY^j/dx^+$ and $\tilde{F}_{+j}(x^+) = d\tilde{Y}^j/dx^+$. More generally, there exist mixed T-dual configurations, in which the D-branes carry both transverse-displacement and electromagnetic waves, all travelling in the same direction $O1$ [26, 27, 29].

That these D-branes are good conformal boundary states follows from the fact that the operators $e^{ip^-X^+}$ have non-singular OPEs with each other, as well as with the operators $\dot{X}^j$. Alternatively, the Dirac-Born-Infeld equations are satisfied trivially because all terms

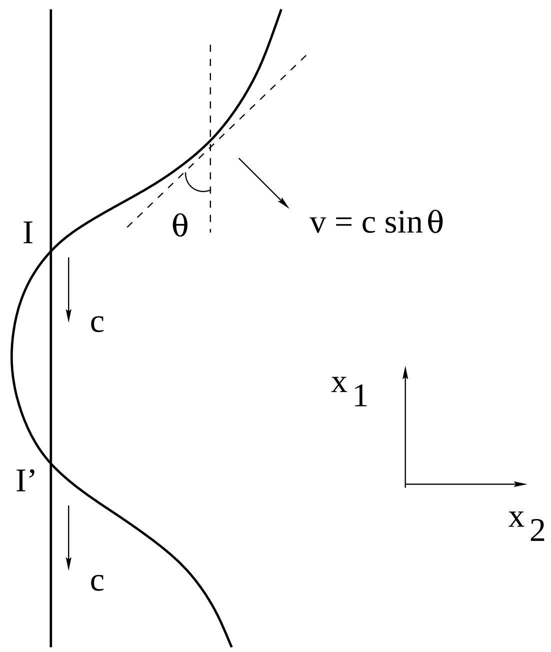

Figure 4: A deformed, twice-intersecting null-scissors configuration, preserving 1/4 of the spacetime supersymmetries. It corresponds to setting $\tilde{Y}^j = 0$ in equation (6.1), with a non-trivial function $Y^2(x^+)$. Both intersection points move at the speed of light in the negative 01 direction.

involving an upper index $\mu = +$ (or lower index $\mu = -$) vanish. Turning on such marginal deformations one can form multiply-intersecting supersymmetric 'scissors', like the ones illustrated in figure 4.

Let us go back now to the straight scissors, on which stretched open strings have been produced in a collision process. As we already argued, this will force the two D-strings to intersect, so that the charged excitations can decay to normalizable massless states. What can happen next is illustrated in figure 5: the two D-strings may join and split as in a standard string-interaction process. The charged excitations can then be converted to travelling waves of the type that was discussed above.

Note that the two configurations of figure 5 are gauge-equivalent when the two D-strings touch. This is most easily seen by combining the transverse-displacement fields in 2×2 matrices $\mathbf{Y}^j$, which transform in the adjoint representation of the gauge group U(2). Then the gauge transformation from the configuration on the the left to the one on the right (for zero shift) can be written as follows:

$$\mathbf{Y}^{2\,\prime} = \sqrt{2}\,\tan\theta \begin{pmatrix} \Theta(x^+)x^+ & 0 \\ 0 & \Theta(-x^+)x^+ \end{pmatrix} = g(x^+)\mathbf{Y}^2 g(x^+)^{-1}\,, \tag{6.2}$$

where $\Theta(y)$ is the Heaviside step function,

$$\mathbf{Y}^2 = \sqrt{2}\,\tan\theta \begin{pmatrix} x^+ & 0 \\ 0 & 0 \end{pmatrix}, \qquad \text{and} \qquad g(y) = \exp\left(i\Theta(y)\sigma^1\right). \tag{6.3}$$

Note also that 'wedge' D-strings, like those on the right-hand side of figure 5, are special cases of the general 1/4 supersymmetric configurations (6.1).

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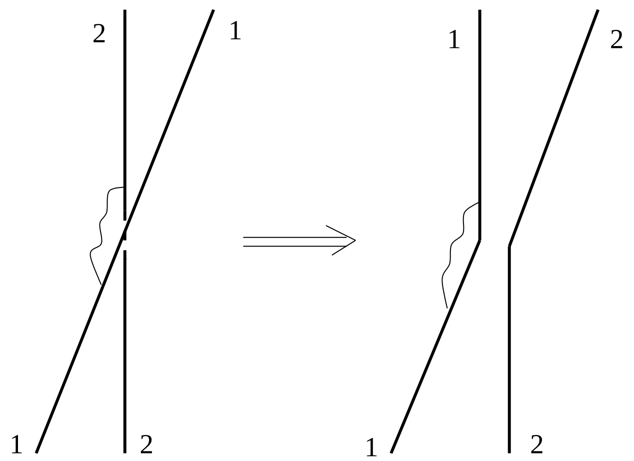

Figure 5: The joining and splitting of D-strings mediated by charged excitations. When the two D-strings touch, the configurations on the left and right are gauge equivalent. Such a process converts the two relatively-boosted strings into a single long string, when space is compactified on a torus.

If the D-brane scissors live on a compact torus, then a non-BPS excitation will keep colliding with the intersection point, or more generally with the supersymmetric travelling waves, until all the momentum p^+ has been carried away through closed-string emission to the bulk. This is the D-brane analogue of the process of Hawking radiation from a nearly-extremal charged black hole (for a recent review see [33]). In the process, the two D-strings will generically interact as in figure 5, to form a single longer string for which the entropy is maximal. This is also consistent with our conclusions of section 5, where we saw that small perturbations of the D-strings, or of the closed-string torus moduli, will force the former to bind into a subthreshold bound state.

Let us now summarize our main point: one-particle excitations can catalyze a condensation of zero modes, and modify drastically the underlying vacuum state. The basic mechanism proceeds through the production of stretched open strings, which force the D-branes to intersect and recombine in a different way. Furthermore, successive incoming excitations will keep modifying the shape of the branes in the near-contact region.

What does this teach us about the physics of the parabolic orbifold and of the null fluxbrane? Recall that these closed-string backgrounds are obtained by identifying points of $\mathbb{R}^{9,1}$ under a Poincaré transformation, which is a combination of the null boost (2.5) and of a translation by $2\pi r$ in a transverse dimension. The resulting geometry describes a circle whose radius is a function of the light-cone time x^+: it starts from infinite size in the distant past, shrinks to a minimum radius r at $x^+ = 0$, and then re-expands to infinite radius in the future. The intersection point of the null scissors is replaced here by a Big-Bang singularity. Furthermore, twisted closed strings share the same properties as stretched open strings in the D-brane scissors: their normalizable physical states have $p^+ = M_\perp = 0$, and only exist in the unshifted case $r = 0$.

Arguing by analogy, we may expect that particles falling towards the cosmological singularity from the asymptotic past can produce pairs of twisted strings, which would then catalyze a condensation of closed-string states. This could modify drastically the nature of the Big Bang singularity. If successive infalling excitations keep changing the physics at the singularity, a statistical treatment of this would be required. Note that this mechanism[7] differs from the strong-gravitational effects discussed recently in references [7, 11, 12, 14]. It is a stringy phenomenon that should dominate for fixed r and p^+, and very weak string coupling. In this limit there will be no formation of black holes [14], and the problem might be amenable to a perturbative treatment. If so, and assuming that the above analogy remains good, the null D-brane scissors could be a simple model for the resolution of a cosmological singularity in string theory.

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A. Particle in constant electromagnetic background

In this appendix we derive the canonical commutation relations for a relativistic charged point particle coupled to a constant electromagnetic background, for comparison with the results of section 4. This is described by the following lagrangian and constraint:

$$L = \frac{1}{2}\dot{X}^\mu \dot{X}_\mu - \frac{1}{2}F_{\mu\nu}X^\mu \dot{X}^\nu \qquad \text{and} \qquad \dot{X}^\mu \dot{X}_\mu + M^2 = 0\,.$$

We restrict the discussion to two space dimensions, so that $\mu, \nu = 0, 1, 2$. From L we can derive the equations of motion

$$\ddot{X}^\mu = -F^{\mu\nu}\dot{X}_\nu\,,$$

and the canonical commutation relations:

$$[X^\mu, \Pi_\nu] = i\delta^\mu_{\ \nu} \qquad \text{with } \Pi_\nu = \dot{X}_\nu + \frac{1}{2}F_{\nu\rho}X^\rho\,.$$

There exist three inequivalent possibilities, according to the sign of $F_{\mu\nu}F^{\mu\nu}$, which we analyze separately.

(i) **Magnetic:** we can set $F_{0i} = 0$ and $F_{12} = -B$. The general solution reads

$$X^0 = x^0 + p^0\tau\,, \qquad \frac{X^1 + iX^2}{\sqrt{2}} = x + ae^{-iB\tau}\,,$$

where the constants of the motion x and a are complex. Canonical quantization implies the following non-zero commutators:

$$[a, a^*] = [x^*, x] = \frac{1}{B} \qquad \text{and} \qquad [x^0, p^0] = -i\,,$$

[7] Ideas similar to the ones advocated here have been also discussed by Fabinger and McGreevy [13], and by Bachas and Nekrasov [34].

while the constraint equation reads:

$$(p^0)^2 = (aa^* + a^*a)B^2 + M^2\,.$$

This leads to the usual spectrum of Landau levels, with a degeneracy proportional to area times $B/2\pi$.

(ii) **Electric:** we can set $F_{01} = E$ and the remaining $F_{\mu\nu} = 0$. The problem can be solved by analytic continuation from the magnetic case. The general trajectory reads:

$$X^2 = x^2 + p^2\tau\,, \qquad \frac{X^0 \pm X^1}{\sqrt{2}} = x^\pm + a^\pm\, e^{\pm E\tau}\,,$$

where $x^\pm$ and $a^\pm$ are real. Canonical commutators imply

$$[a^-, a^+] = [x^+, x^-] = -\frac{i}{E}\,, \qquad [x^2, p^2] = i\,, \quad \text{rest} = 0\,.$$

The problem is equivalent to an inverted harmonic oscillator, so that the constraint

$$(a^+a^- + a^-a^+)E^2 + (p^2)^2 + M^2 = 0$$

does not admit any normalizable solutions.

(iii) **Null:** here we choose $F_{+2} = f$ with the remaining components zero. The general classical solution reads

$$X^+ = x^+ + p^+\tau\,, \qquad X^2 = x^2 + p^2\tau + fp^+\frac{\tau^2}{2}\,, \qquad X^- = x^- + p^-\tau + fp^2\frac{\tau^2}{2} + f^2p^+\frac{\tau^3}{6}\,.$$

At time $\tau = 0$ we have $X^\mu(0) = x^\mu$, and $\Pi_\mu(0) = p_\mu + \frac{1}{2}F_{\mu\rho}x^\rho$. Thus the canonical commutation relations imply:

$$[x^\mu, p^\nu] = i\,\eta^{\mu\nu}\,, \qquad [p^-, p^2] = if\,, \qquad \text{rest} = 0\,.$$

These agree with the relations (4.18) if we set $2\alpha' = 1$ (so as to normalize the string's kinetic energy correctly). Furthermore, the mass-shell condition has the standard form,

$$p^\mu p_\mu + M^2 = 0\,.$$

Shifting p^- as in (4.19), maps the problem to that of a point particle moving in a linear potential $2p^+fx^2$. This has normalizable solutions only when $p^+ = 0$, in which case the linear potential vanishes. These normalizable solutions describe massless charged particles propagating at the speed of light in the direction normal to the electric and magnetic fields.

We can understand this phenomenon by a limiting procedure. Start with a pure magnetic field, $F'_{+2} = -F'_{-2} = B/\sqrt{2}$, and perform a boost in the 01 direction to a frame where $F_{\pm 2} = e^{\pm\xi}F'_{\pm 2}$. The null configuration is obtained in the limit $\xi \to \infty$, $B \to 0$ with $F_{+2} \equiv f$ held finite and fixed. States with energy $E' \sim nB$ in the original problem undergo an infinite boost, which sends $p^+ \to 0$ with p^- and the energy E held fixed. Transverse momentum in the primed frame must also be scaled to zero like $\sqrt{B}$, or else the energy in the limit would diverge.

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